Calculus AB
4-2
Area
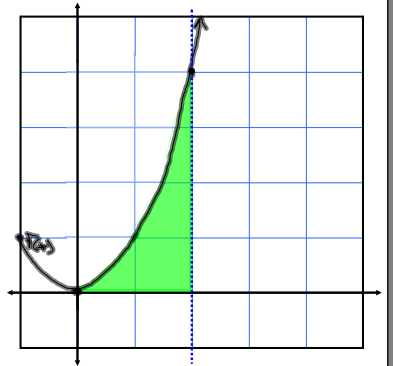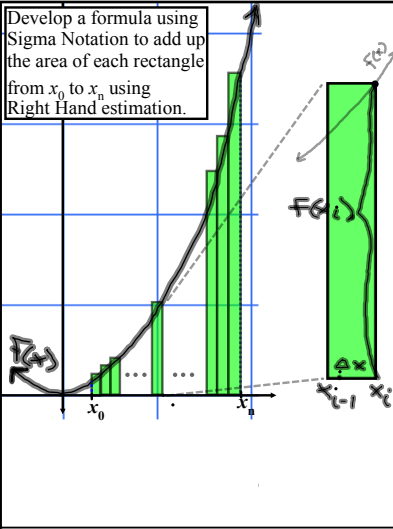

Find the area bounded by the x-axis and $f(x) = x^2$ between 0 and 2.

Develop a formula using Sigma Notation to add up the area of each rectangle from x_0 to x_n using Right Hand estimation.

$f(x)$

$F(x_i)$

Δx

x_0 ... ... x_n

How many rectangles are used to estimate the area?

How do I calculate the area of any generic rectangle?

$$F(x_i)\Delta x$$

What is the formula using Sigma to add all the areas?

$$\sum_{i=1}^{n} F(x_i)\Delta x$$

How do I calculate Δx?

$\Delta x = x_i - x_{i-1}$

if all equal $\Delta x = \dfrac{x_n - x_0}{n}$

$x_{i-1} \quad x_i$

Find the area bounded by the x-axis and $f(x) = x^2$ between 0 and 2 using Right Hand estimation with 4 rectangles.

$x_n = 2$
$x_0 = 0$
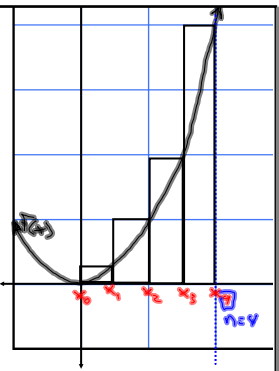$n = 4$

$\boxed{\dfrac{15}{4}}$

$$\sum_{i=1}^{4} F(x_i)\Delta x$$

$$\sum_{i=1}^{4} (x_i)^2 \left(\frac{2-0}{4}\right) \quad \Delta x = \frac{1}{2}$$

$x_1 = \frac{1}{2}$
$x_2 = 1$
$x_3 = \frac{3}{2}$
$x_4 = 2$

$x_1 \Delta x \quad x_2 \Delta x \quad x_3 \Delta x \quad x_4 \Delta x$

$f(x)$

$x_0 \ x_1 \ x_2 \ x_3 \ x_4$ $n=4$

$$\sum_{i=1}^{4} (x_i)^2 \Delta x = \left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right) + (1)^2\left(\frac{1}{2}\right) + \left(\frac{3}{2}\right)^2\left(\frac{1}{2}\right) + 2^2\left(\frac{1}{2}\right)$$

$$\frac{1}{8} + \frac{1}{2} + \frac{9}{8} + 2 = \boxed{\frac{15}{4}}$$

Develop a formula using Sigma Notation to add up the area of each rectangle from x_0 to x_n using **Left Hand** estimation.

$f(x)$

$F(x_{i-1})$

Δx

x_0 ... ... x_n

$x_{i-1} \ x_i$

What is the formula using Sigma to add all the areas?

$$\sum_{i=1}^{4} F(x_{i-1})\Delta x$$

Find the area bounded by the x-axis and $f(x) = x^2$ between 0 and 2 using Left Hand estimation with 4 rectangles.

$x_n \quad x_0$

$\boxed{\dfrac{7}{4}}$

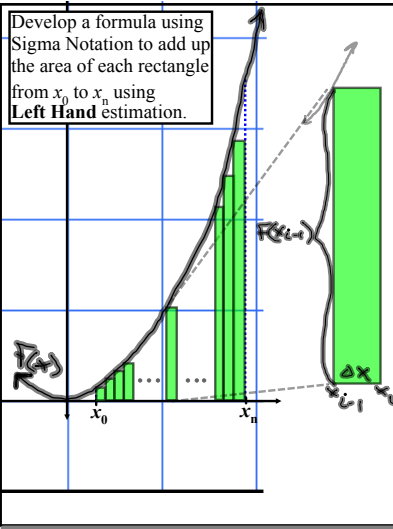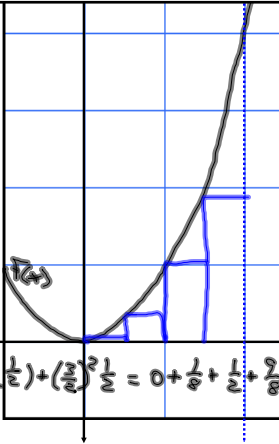$n = 4 \qquad \Delta x = \dfrac{2-0}{4} = \dfrac{1}{2}$
$n-1 = 3$

$x_0 = 0$
$x_1 = \frac{1}{2}$
$x_2 = 1$
$x_3 = \frac{3}{2}$
$x_4 = 2$

$$\sum_{i=1}^{4} (x_{i-1})^2 \Delta x$$

$$= (0)^2\frac{1}{2} + \left(\frac{1}{2}\right)^2\frac{1}{2} + (1)^2\left(\frac{1}{2}\right) + \left(\frac{3}{2}\right)^2\frac{1}{2} = 0 + \frac{1}{8} + \frac{1}{2} + \frac{9}{8} = \frac{7}{4}$$

$f(x)$

Slide 1
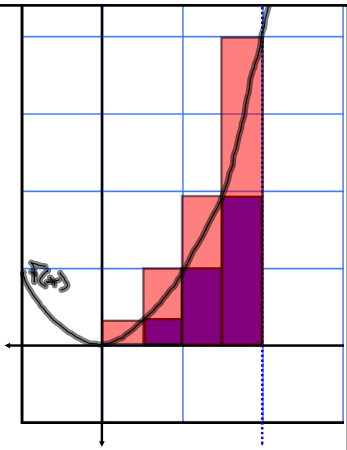
Compare Right Hand estimation to Left Hand estimation.

$\frac{7}{4}$ $\frac{15}{4}$

≈ 4

Find the average

$\frac{\frac{7}{4} + \frac{15}{4}}{2} = \frac{\frac{22}{4}}{2}$

$\frac{\frac{11}{2}}{2} = 4$

Slide 2

Slide 3

Develop a formula using Sigma Notation to add up the area of each rectangle from x_0 to x_n using **Midpoint** estimation.
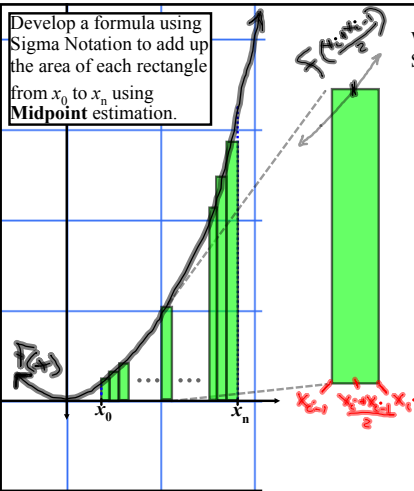
What is the formula using Sigma to add all the areas?

$$\sum_{i=1}^{n} F\left(\frac{x_i + x_{i-1}}{2}\right) \Delta x$$

Slide 4

Find the area bounded by the x-axis and $f(x) = x^2$ between 0 and 2 using Midpoint estimation with 4 rectangles.

$$\sum_{i=1}^{4} F\left(\frac{x_i + x_{i-1}}{2}\right) \Delta x$$

$\Delta x = \frac{1}{2}$

$x_0 = 0$

$x_4 = \frac{1}{4}$

$x_1 = \frac{1}{2} \rightarrow \frac{3}{4}$

$x_2 = 1 \rightarrow \frac{5}{4}$

$x_3 = \frac{3}{2} \rightarrow \frac{7}{4}$

$x_4 = 2$

$\left[\left(\frac{1}{4}\right)^2 + \left(\frac{3}{4}\right)^2 + \left(\frac{5}{4}\right)^2 + \left(\frac{7}{4}\right)^2\right] \frac{1}{2}$

$2\frac{5}{8} = \frac{21}{8}$